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Photonic materials in circuit quantum electrodynamics

Iacopo Carusotto[1], Andrew A. Houck[2], Alicia J. Kollár[3,4], Pedram Roushan[5], David I. Schuster[6,7] and Jonathan Simon[6,7] ✉

Photonic synthetic materials provide an opportunity to explore the role of microscopic quantum phenomena in determining macroscopic material properties. There are, however, fundamental obstacles to overcome — in vacuum, photons not only lack mass, but also do not naturally interact with one another. Here, we review how the superconducting quantum circuit platform has been harnessed in the last decade to make some of the first materials from light. We describe the structures that are used to imbue individual microwave photons with matter-like properties such as mass, the nonlinear elements that mediate interactions between these photons, and quantum dynamic/thermodynamic approaches that can be used to assemble and stabilize strongly correlated states of many photons. We then describe state-of-the-art techniques to generate synthetic magnetic fields, engineer topological and non-topological flat bands and explore the physics of quantum materials in non-Euclidean geometries — directions that we view as some of the most exciting for this burgeoning field. Finally, we discuss upcoming prospects, and in particular opportunities to probe novel aspects of quantum thermalization and detect quasi-particles with exotic anyonic statistics, as well as potential applications in quantum information science.

Experiments with light provide one of the backbones around which the scientific community has built our understanding of the bizarre and beautiful implications of the laws of quantum mechanics. Historically, most experiments with photons — the quanta of the electromagnetic field — have explored elementary processes involving the generation, manipulation and detection of a few such particles[1]. The entanglement between a pair of photons, for example, provided the pivotal evidence supporting quantum mechanics over hidden variable theories[2,3].

In the last decade, the situation changed with the advent of quantum fluids of light[4]: under suitable conditions, photons inherit an effective mass from the structure confining them, and collide with one another due to the nonlinear optical response of the structure. Together, these properties enable macroscopic ensembles of photons to exhibit collective behaviours akin to ordinary fluids. As compared to a standard description of light in the language of nonlinear and quantum optics, thinking in terms of a gas or fluid of interacting bosonic particles offers new insights and unexpected perspectives gleaned from the world of condensed matter.

While this adventure had its first breakthroughs in the infrared and visible domains using exciton polaritons in semiconductor microcavities[5,6] and Rydberg polaritons in atomic gases[7], exciting new possibilities have recently emerged from microwave photons in superconducting quantum circuits. Although these circuits were originally developed for quantum computing purposes, the same features — single-photon non-linearities[8], long coherence times[9,10] and excellent scalability[11] — are essential ingredients for realizing synthetic photonic quantum materials. As such, where most nonlinear and quantum optics studies until the late 2000's focused on the semi-classical collective dynamics of many photons or the quantum dynamics of few photons, the circuit quantum electrodynamics (QED) platform enables studies of the quantum dynamics of many strongly correlated photons — the regime where microwaves behave as a novel photonic material. Earlier reviews exploring the promise of this direction (but not its realization) can be found in[4,11–14].

This Review Article will focus on materials made of light that are built and manipulated in the circuit QED platform. This new approach dramatically broadens the scope of models and physical phenomena that are experimentally addressable beyond what is accessible in traditional materials composed of atoms in solids and liquids, electrons in solids, or protons and neutrons in nuclei. Nonetheless, the properties of synthetic materials are governed by the same mechanisms that control traditional materials, namely competition between interactions, kinetic energy, temperature and, in the case of driven quantum-material properties, dephasing.

Quantum matter by the numbers

A comparative summary of the characteristic energy scales in traditional materials versus their synthetic counterparts in photonic and cold atom platforms is provided in Fig. 1. What is clear is that, from the perspective of interactions compared with decoherence, photonic quantum materials now have the potential to support comparable large-scale entanglement to other platforms. Beyond this, photonic materials offer dramatic advantages over traditional solid-state systems, including new probes with direct quantitative access to microscopic observables and a fast experimental repetition rate that enables accumulation of statistics on complex many-body quantum correlators.

In addition to these technical advantages, the many-body physics accessible in photonic systems also offers exciting new perspectives, particularly in the ease of introducing driving and dissipation. Until very recently, the general understanding was in fact that photonic platforms are more lossy and thus less 'quantum' than their atomic and electronic counterparts, and that this was an ultimate limitation precluding strongly correlated many-body states of light. This perception originated at a time when the available nonlinearities were either weak or inextricably tied to strong dissipation, as happens in most optical material. With the advent of circuit QED, many of the extrinsic decoherence channels were tamed, and selective dissipation can now be re-introduced as a powerful feature of the platform. Indeed, central challenges today have more to do with the design

[1]INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, Trento, Italy. [2]Princeton University, Princeton, NJ, USA. [3]University of Maryland, College Park, MD, USA. [4]The Joint Quantum Institute, College Park, MD, USA. [5]Google Inc, Santa Barbara, CA, USA. [6]University of Chicago, Chicago, IL, USA. [7]The James Franck Institute, University of Chicago, Chicago, IL, USA. ✉e-mail: simonjon@uchicago.edu

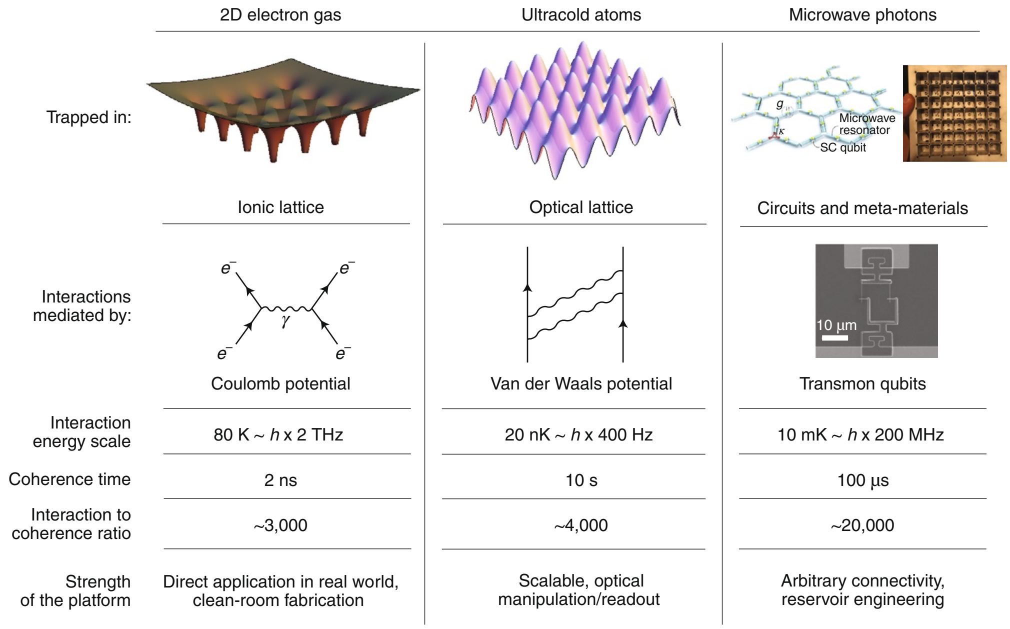

Fig. 1 | A comparison of quantum matter platforms. Figures of merit for 2D electron gases are calculated for high-mobility AlGaAs/GaAs heterostructures, based upon mobilities from ref. [160] and typical Coulomb energy in the lowest Landau level at a magnetic field of 2.5 T (ref. [161]). For cold atoms, typical numbers come from ref. [15]. For microwave photons, numbers come from refs. [9,42]). Note that, because electrons and atoms do not decay quite as photons do, the numbers for 2D electron gases reflect disorder, while those for atomic gases in optical lattices reflect atom losses due, for example, to background gas collisions, lattice-induced heating, and inelastic scattering. *g*, the qubit–cavity coupling strength; *κ*, the cavity linewidth; SC, superconducting.

of suitable dissipation and pumping schemes that are able to selectively generate specific quantum phases of the photonic material.

Routes to ordering

A comparison with traditional condensed matter systems is useful to put this problem in a wider perspective: electronic systems are typically prepared at or near to thermal equilibrium with their solid-state environment, which is itself kept at the required low temperature through standard cryogenic techniques. The quantum state then naturally arises as the low-temperature phase of the many-body system under investigation. The preparation stage is slightly more subtle in cold atom systems, where the dominant approach to building many-body states relies upon initial preparation of a low-entropy Bose–Einstein condensate by laser and evaporative cooling of the atoms, and then adiabatic variation of the system Hamiltonian, for example by introducing an optical lattice[15,16]. If a suitable adiabatic path is chosen[17,18] and the process is carried out slowly enough, the (unentangled) initial state is adiabatically converted into the (highly entangled) ground state of the final Hamiltonian (Fig. 2a). The key ingredients for this approach are (1) a way to remove entropy from the system and (2) the ability to vary the system Hamiltonian at a suitable speed. While this approach is possible with quantum circuits, it has thus far proven technically challenging.

Shortly after the first proposals of strongly correlated photonic matter[19–21] and in parallel to related studies in non-equilibrium Bose-Einstein condensates of photons and polaritons[4,22–27], the need to address the unique driven-dissipative characteristics of optical systems was recognized by the community[28]. The next generation of proposals[29,30] and experiments[31] relied upon spectroscopically resolved excitation of the system, one particle at a time (Fig. 2b), akin to laser excitation of an atom or molecule: the fact that this process requires detailed knowledge of the many-body spectrum of the system and is sensitive to particle loss, disorder and other perturbations leads again to the undesired perception that photons are unfavourable for quantum materials science.

To circumvent these challenges, a promising modern approach to explore photonic quantum materials relies upon continuous coupling of the system to non-Markovian reservoirs that compensate particle loss whilst simultaneously cooling the system. The resulting quantum dynamics must then be understood in a driven-dissipative many-body paradigm, potentially reaching a dynamical steady-state rather than thermodynamic equilibrium. Years of theoretical and experimental advances have revealed that these non-equilibrium systems provide unique physics of their own: from time-crystals[32,33] to light-induced superconductivity[34] and beyond, far-from-equilibrium dynamics displays a much richer phenomenology than its quasi-equilibrium counterpart[35]. Ramping up in complexity, engineered coupling to the environment (frequency-dependent non-Markovian damping) has been shown to provide a powerful tool for controlled entropy removal[36–41], allowing driven-dissipative systems to be continuously cooled towards correlated many-body steady-states[42] (Fig. 2). Such entropy manipulation will be a central thread in this review.

Basics of circuit QED for photonic quantum matter

Circuit QED is a framework that uses superconducting qubits and cavities to manipulate the quantum state of microwave photons[43,44]. The unique versatility of this platform stems from the flexibility in the design of the system geometry and of the light-matter coupling, both of which can be engineered using conventional lithography. This versatility allows one to change the character of the Hamiltonian studied without significant changes to the materials or the experimental apparatus. The essential ingredients of the circuit-QED toolbox for photonic quantum materials are the superconducting resonator, which provides a home for non-interacting photons, and the 'transmon' qubit[45], which acts as the lattice sites in which photons reside and strongly interact with one another.

The transmon, as shown in Fig. 3, is a capacitively shunted Josephson junction: it may be understood as an inductor–capacitor

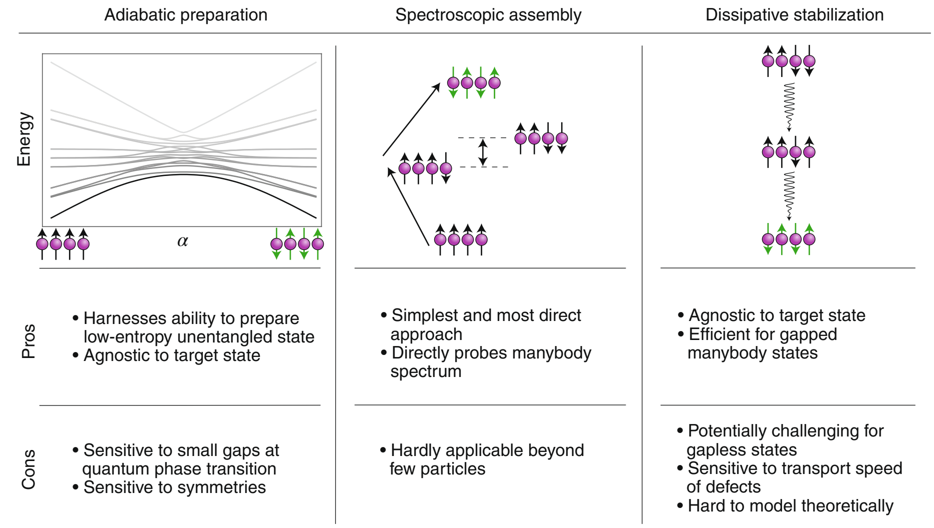

Fig. 2 | Assembling quantum matter. A summary of the various approaches to assembly and stabilization of quantum materials, using the antiferromagnet, highlighted with green arrows, as a paradigmatic target state that all approaches aim to reach. Adiabatic preparation begins with the easily prepared, unentangled/uncorrelated paramagnetic state, and slowly varies the Hamiltonian (via a tuning parameter α) such that the ground state (thick black curve) is smoothly converted from the paramagnet into the antiferromagnet. Spectroscopic assembly again begins with the paramagnet, and converts it into the antiferromagnet through sequential coherent pulses that energetically resolve the intermediate states. Dissipative stabilization begins in any state, and an engineered dissipation process continuously pumps the system towards the desired target state.

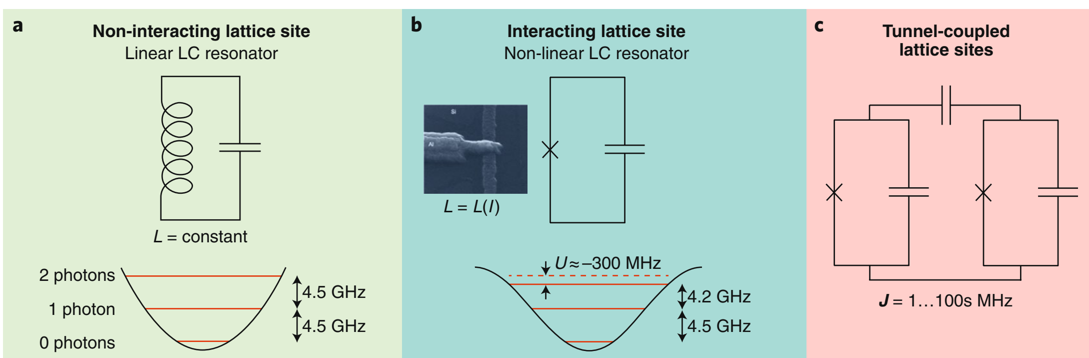

Fig. 3 | The circuit QED toolbox for materials. The microwave photon is the essential constituent of the photonic quantum materials explored in this review: it acts as the basic constituent from which the material is made, while interactions between photons are mediated by the nonlinear electromagnetic response of the underlying medium. **a**, A resonator composed of an inductor and a capacitor can trap and hold an arbitrary number of microwave photons at frequencies in the few-gigahertz range. As in a textbook quantum harmonic oscillator, in such a resonator the photons do not interact. This reflects in the spacing between neighbouring energy levels — that is, photon number states — being independent of the number of photons residing within the resonator. **b**, To introduce interactions between the photons, the inductor is replaced with a strongly nonlinear element such as a capacitively shunted Josephson junction. This device provides an inductance which depends upon the current flowing through it, and thus creates a non-uniform energy-level spacing for this 'transmon' qubit. In typical set-ups, this corresponds to a photon–photon interaction energy U of a few hundred megahertz. **c**, To enable motion of photons between lattice sites, neighbouring transmons or resonators are coupled through a coupling element (for example, a capacitor) that allows photon tunnelling from one resonator to another, with hopping energy J tunable across the interaction energy. Proper fabrication can suppress photon/qubit decay down to $2\pi \times 10$ Hz/1 kHz respectively[10,162], with comparable dephasing.

(LC) resonator whose inductance (derived from the Josephson effect) depends strongly upon the current flowing through it. This results in a photon-blockade-like[46] phenomenon, where the energy of the $0 \rightarrow 1$ transition from the zero- to the one- photon state (typically $\omega_{01} \approx 2\pi \times 5$ GHz) in the LC circuit is substantially different than that of the $1 \rightarrow 2$ transition adding a second photon. This energy difference, $U \approx 2\pi \times -300$ MHz, may be viewed as a strong attractive interaction of energy $\hbar U$ between photons residing in the same transmon. That is, photons on the same lattice site feel each others presence in a time $T_{coll} \approx |U|^{-1}$.

If transmons act as the sites of a lattice and their intrinsic nonlinearity provides effective attractive interactions between photons in each site, the next thing we need is to induce tunnelling between the sites. In the simplest scheme, this is achieved via a capacitive coupling between the transmons, which induces a coherent tunnelling between lattice sites of order $J \approx 2\pi \times 1$–100 MHz. When sites are arranged in periodic lattice geometries, photonic analogs of the crystalline lattice of solid-state materials, or the periodic optical potential of optical lattices for ultracold atoms, are obtained. Beyond what is typically possible in atomic systems, the lattice geometry of circuit QED systems can be designed bottom-up with great flexibility[47,48] and the tunnelling amplitude can be externally tuned in real time in both magnitude and phase[49,50].

In a generic material, ordering among the constituent particles arises from the interplay of kinetic and interaction energies. In photonic systems, dissipation and pumping introduce a further energy scale related to the decoherence rate γ, so that the 'quantumness' of a photonic material can be roughly characterized by the number of (entanglement-producing) collisional events a particle in the photonic material of density $n_{\rm phot}$ per lattice site (typically of order unity) undergoes before interacting with the environment and collapsing the many-body wavefunction of the system, $N_{\rm coll} \approx n_{\rm phot}|U|/\gamma$.

This review is focused on the strongly interacting regime $N_{\rm coll} \gg 1$, where the particles have time to become strongly entangled before the many-body wavefunction collapses due to decoherence. This regime is to be opposed to a semiclassical regime where the collective action of a large number of particles is required to see an appreciable effect of interactions and the dynamics is thus accurately described with a (classical) mean-field theory analogous to the Gross–Pitaevskii regime of a dilute Bose–Einstein condensate[51].

While typical nonlinear optics experiments deal with this semiclassical regime, strong efforts are presently underway to realize strong interactions between optical photons[7]. The present state of the art in optical quantum photonics follows several thrusts: polariton lattices in microcavities embedding quantum wells[52,53] with interactions approaching the quantum regime[54,55]; free-space photons coupled to Rydberg atoms, which have reached the quantum regime, with a few collisions per photon lifetime[56,57]; and cavity photons coupled to Rydberg atoms, with the possibility of more collisions and complex many-mode dynamics[58–60]. By comparison, the circuit QED experiments under consideration here already reach beyond 10^4 collisions per photon lifetime, positioning the platform deep in the quantum regime and opening the way to studies of many-body physics of strongly interacting particles.

In contrast to traditional condensed matter where experiments typically probe macroscopic quantities, a further important asset of circuit materials is the possibility of simultaneous, high fidelity (>99%) read-out of occupation of the individual lattice sites, for example, using the transmons' photon-number-dependent dispersive shift[8,61,62]. This provides a spatio-temporally resolved readout of observables and correlation functions akin to an atomic quantum gas microscope[63,64], with the crucial advantage that one is not restricted to measurements in the occupation basis. For instance, observables involving coherences between different number states can be measured by applying on-site qubit rotations[65] prior to measurement.

Strongly interacting lattices

The first decade or so of circuit QED experiments focused primarily on the nonlinear quantum dynamics of devices involving at most a few cavity modes coupled to qubits[43,66–69]. While this required substantial developments in microwave quantum technology, the unprecedented nonlinearity and coherence time that was eventually achieved have allowed for demonstrations of textbook quantum optical effects with unique clarity[8,70–72]. In light of these remarkable successes, the focus of the community is now shifting towards systems of many cavities and/or many qubits, whose physics is dominated by many-body entanglement with the potential to explore an exponentially larger Hilbert space[4,11,13,14,44,73].

Early experiments in this direction include a delocalization–localization transition explored in a two-site Hubbard model[74] and spectroscopy and cooling into few-body eigenstates of a three-site Hubbard model[31]. In the former experiment, inspired by theoretical work in ref. [75], a large coherent state was injected into a single site of the Hubbard chain and coherent wave-like tunnelling dynamics observed until photon loss made the interaction energy comparable to the tunnelling energy and the photons were localized by a variant of the well-known self-trapping mechanism of Bose–Einstein condensates[76,77]. In the latter experiment, precise a-priori knowledge of the few-particle energy spectrum enabled spectroscopically resolved population of target eigenstates, where coupling to a lossy cavity enabled engineering of the decay dynamics to lead to autonomous stabilization of a desired few-body state.

As experimental capabilities have improved, it has become possible to prepare arrays of qubits corresponding to low-disorder Hubbard chains[42,65] and start investigating one- and two-body physics[78,79], opening the door to exploration of myriad proposals for quantum many-body physics of photons on a lattice[4,13,14,73]. A central question that arose in this context, and became an intellectual driver of the field, is: given the ability to realize a desired photonic Hamiltonian, how does one populate it with photons and drive it into a particular many-body state or material phase?

This challenge is of course not unique to microwave photons and naturally arises in any synthetic quantum system that is not in thermal contact with its environment. In the case of ultracold atoms in optical lattices, the solution was to (1) make use of laser and evaporative cooling techniques to prepare a Bose–Einstein condensate of weakly interacting atoms and, then, (2) slowly change the system Hamiltonian (by the introduction of optical potentials, magnetic fields, two-body interactions, and so on) to reach a desired target Hamiltonian. Stage (1) removes all/most entropy from the system and dumps it into scattered optical fields and the motion of untrapped atoms, thereby preparing with high fidelity the ground state of the weakly-interacting many-body Hamiltonian; stage (2) then adiabatically (and thus isoentropically) converts this relatively simple ground state into the strongly correlated ground state of the target Hamiltonian. This method, based on global control of system parameters in systems of macroscopic size, was tremendously successful for cold atoms, enabling exploration of the superfluid to Mott insulator quantum phase transition[16], paramagnet to antiferromagnet transition[15], and more recently, studies of the Fermi-Hubbard model[80], all while avoiding the need to cool the system at any time other than the very beginning, when the atomic gas is weakly interacting and evaporation most effective.

While cold-atom systems employ global knobs that simultaneously control a huge number of sites with utmost precision, tuning tunnel couplings and on-site energies in a quantum circuit typically requires local control on each site. As a result, the procedure is likely sensitive to the details of the disorder, and imposes substantial technical limitations. Whereas this approach has nevertheless led to several breakthrough experiments (in particular the one in ref. [50] that is discussed in detail in the section 'Topological lattices' below), these difficulties typically restrict its efficiency to small systems with few photons. Rather than such a head-on confrontation with technical challenges, a more promising strategy is to directly engineer reservoirs for both particle injection and entropy removal in the strongly interacting regime. Several works[36–41,81–84] have in fact proposed routes to 'dissipatively stabilize' quantum many-body phases in the steady state by means of a continuous preferential injection of particles at particular energies and removal of particles at other energies, resulting in cooling of the quantum many-body system. In contrast to the reversible nature of coherent driving, both in its original formulation[29] and in more sophisticated two-photon versions[83,85], the preference for photon injection versus removal in different frequency windows requires breaking of reversibility/unitarity and thus implies a coupling to an external reservoir. In circuit QED platforms such reservoirs are practically realized either by exploiting the band gap of photonic waveguides[86] or using the frequency-selectivity of resistor–inductor–capacitor (RLC) resonators involved in the photon injection process[42].

This frequency-selectivity-based approach to entropy removal relies upon an energetic distinction between 'low-energy' and

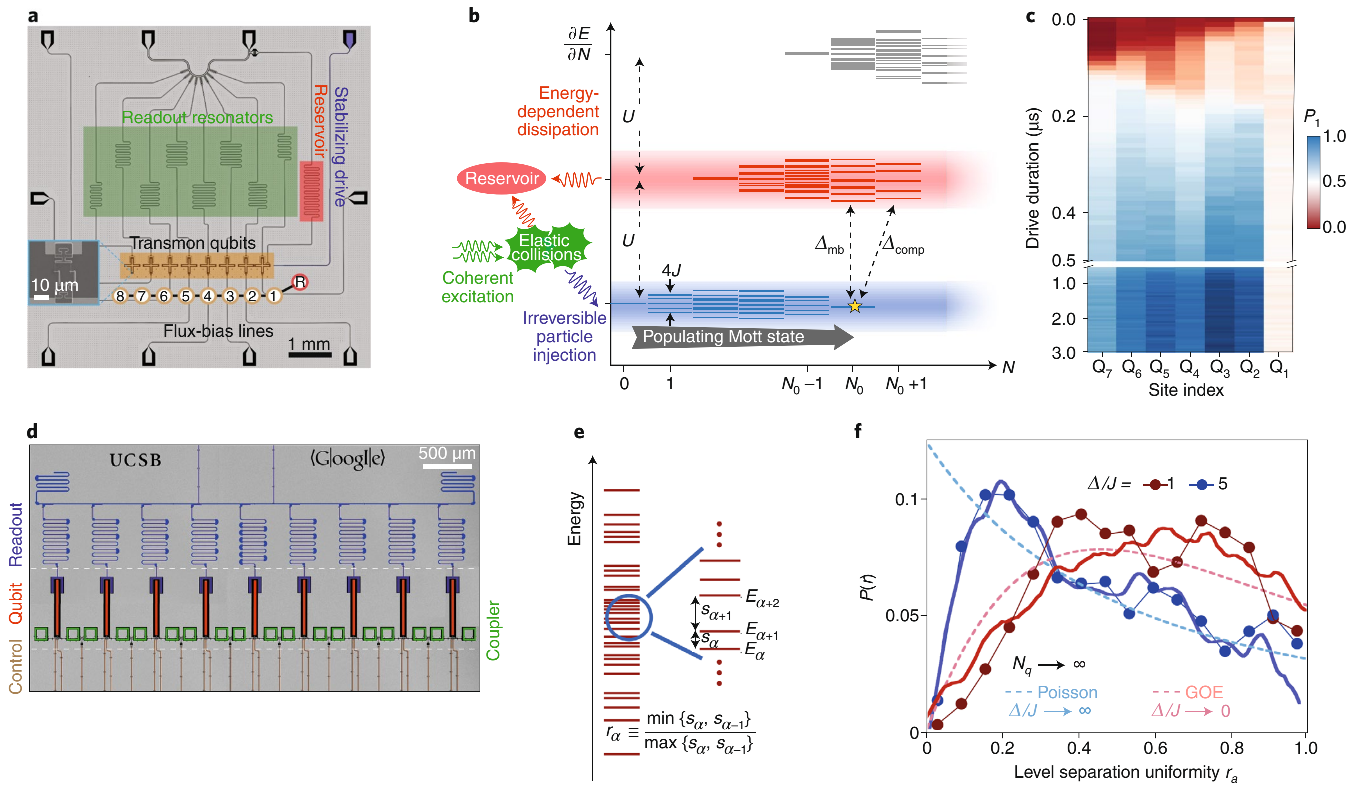

Fig. 4 | Strongly correlated photonic matter in circuit QED. Summary of two recent experiments demonstrating **a**–**c**, a Mott-insulator state of photons[42] and **d**–**f**, interaction-induced localization effects[65]. **a**,**d**, Micrograph of the devices used in the experiments: lattice sites are arranged in a 1D chain and tunnelling between sites is mediated by capacitors (**a**) or flux-tunable couplers (**d**). The occupation of each lattice site is measured through frequency-multiplexed readout resonators. In **a**, photons are irreversibly injected into the system through an engineered reservoir, while in **d**, the coherent evolution starting from a suitably factorized initial state is followed in time. **b**, Schematic diagram of the many-photon energy levels involved in the preparation of the Mott-insulator state. The combination of coherent excitation and parametric scattering processes (green cloud) irreversibly injects photons in a limited frequency window roughly corresponding to the non-interacting photon band (blue). This frequency-dependent incoherent pumping assembles the target Mott state one photon at a time, until the lattice is full and it is no longer possible to inject extra photons without traversing the compressibility gap Δ_{comp} to the next Hubbard band (red). States of this band can also rapidly dissipate into the same reservoir modes used for pumping (red ellipse), thus providing evaporation of high-energy particles. We plot with for $U > 0$ in analogy with cold atoms and electrons — transmons in fact exhibit $U < 0$, but this does not impact the physics. Note that adiabatic assembly is sensitive to the fixed-particle-number manybody gap Δ_{mb}, not the related (but non-identical) particle injection compressibility gap Δ_{comp}. $\partial E/\partial N$, energy required to inject an additional photon into a system containing N photons. **c**, Temporal evolution of the preparation process of a Mott state starting from an empty system: the reservoir (at right, Q_1) progressively (right to left) fills the lattice sites Q_{2-7} with photons as time advances (downwards). The color scale indicates the unit-occupancy probability P_1 of each lattice site: the filling front moves with a light cone away from the reservoir, and even reflects off of the far edge of the system. Within a few microseconds, the photon fluid has reached a steady state with near-unity (just below 90%) occupancy. **e**, Scheme of the energy levels of a generic many-body quantum system. Localization phenomena are related to the distribution of the energy-level spacing ratio r_α, defined in terms of the energies of the eigenstates E_α and their differences s_α. **f**, The measured histogram $P(r)$ for disorder/tunnelling ratio $\Delta/J = 1$ and 5. The dashed lines indicated the Poisson $P_{Poisson}$ and the Gaussian orthogonal ensemble P_{GOE} distributions that are expected for $P(r)$ in the two limiting cases of ergodic and many-body localized phases in the thermodynamic limit. The solid lines are numerical simulations for the chain of 9 qubits used in experiments. N_q, number of sites. Figure reproduced with permission from: **a**–**c**, ref. [42], Springer Nature Ltd.; **d**–**f**, ref. [65], AAAS.

'high-energy' states: while the former are quickly refilled to compensate for unavoidable losses, the latter are effectively blocked by the sizable frequency mismatch to the injection RLC. This implements, in the driven-dissipative context, the concept of incompressibility. To achieve this, it exploits the many-body energy gap present above the ground state of many strongly correlated phases of matter, including Mott insulating and fractional quantum Hall states: hole-like states that accidentally form below the many-body energy gap because of losses are quickly refilled, while extra particles cannot be injected above the many-body gap.

This idea underlies the first demonstration of strongly correlated photonic matter, consisting of a Mott insulator of photons[42], realized in an array of eight capacitively coupled transmon qubits, dissipatively pumped by a two-photon drive with entropy dumped into a narrow-band RLC resonator as shown in Fig. 4a. Note that the high fidelity of the resulting Mott state is not a single-site effect but results from many-particle dynamics of the full lattice: the dissipative pumping (Fig. 4b) occurred only at the rightmost end of the chain, allowing for exploration of the gradual growth of the crystalline steady-state at unit filling as new particles are injected and the system equilibrates with the particle-reservoir as displayed in Fig. 4c.

Whereas the quantum state of matter realized in this experiment — the Mott insulator — is a well-known concept in many-body physics, one can reasonably expect that the driven-dissipative nature of the system modifies its properties, adding temporal fluctuations and perturbing its relaxation towards its stationary state.

It is even interesting to ask if the thermal-reservoir coupling in solid-state Mott insulators perturbs their properties.

To these ends, in recent years substantial theoretical effort has been devoted to studying non-equilibrium phase transitions[40,41,73,75,82,84,87–94] and the fluctuations and effective heating mechanisms induced by pumping and dissipation[95–99]; nonetheless the overall understanding of strongly interacting driven-dissipative many-body systems remains less complete than that of isolated systems at or near equilibrium. From a numerical perspective, substantial efforts are still ongoing to develop techniques able to deal with the non-perturbative interplay of hopping, interactions, driving and dissipation[40,100–105].

Another interesting challenge is how to study gapless phases, where dissipative stabilization is more challenging. Beyond weakly interacting non-equilibrium Bose–Einstein condensates[4,22,23], strongly interacting many-body localized states are of particular interest, where disorder broadens the energy spectrum, removing gaps, and also inhibits particle diffusion[106], further complicating the equilibration process.

Rather than attempting to stabilize the ground state of such many-body localized Hamiltonians, the authors of ref. [65] adopt a unique quantum-optics-inspired approach to probing many-body localization: they directly extract the frequency difference between pairs of eigenstates of a disordered Hubbard chain of transmons (Fig. 4d) by looking at quantum beats in the time evolution of suitable one- and two-body observables. In this way, the clear signature of localization appearing in the resulting few-body spectrum is the absence of level repulsion in the statistics of system gaps (Fig. 4d–f). Subsequent work[107] explored arrested relaxation of a density-wave state by disorder, disentangling single- and many-body localization effects through full quantum-state tomography of the ten-lattice-site system.

To date, strongly correlated photonic materials have been explored exclusively in one dimensional (1D) lattices, whose connectivity is a quite trivial one, but another strength of the circuit QED platform is the ability to engineer lattices with more complex connectivities. In the following sections we describe the menagerie of fascinating topological, flat-band and non-Euclidean lattices that are presently under exploration, and discuss their prospects for integration with strong photon–photon interactions.

Topological lattices

From the quantum Hall effect to topological insulators[108], some of the most fascinating properties of solids arise due to band structures that are knotted in suitable abstract spaces. Fundamentally, these effects result from Lorentz forces on electrons in magnetic fields and/or spin–orbit coupling effects in the material, which introduce a handedness in the material's properties.

Exploring such phenomena in the circuit QED platform was initially hindered by the fact that photons are charge-neutral objects, and as such do not experience a Lorentz force in the presence of a magnetic field. To overcome this difficulty, in close connection with simultaneous efforts in the field of neutral atoms[109], an intense effort has been devoted to the realization of synthetic magnetic fields for photons and, more generally, to introduce ideas of topological quantum matter into optics. This has given rise to the new field of topological photonics[110]: beginning with the pioneering theoretical insight in ref. [111] and the breakthrough observation of topologically protected chiral edge states in photonic crystals[112], this field has grown into a new area of optics with numerous applications.

The first proposals for microwave photons in circuits date back to ref. [113], where ferrimagnetic circulator elements were proposed to break time-reversal symmetry and generate topologically non-trivial bands. Soon after, the idea of modulating the site frequencies and/or the hopping amplitudes in time was put forward[114]. These proposals rely on the so-called Peierls substitution, a formal connection between the magnetic field in continuum systems and tunnelling phases in their lattice counterparts: the effective magnetic flux through a plaquette is proportional to the net tunnelling phase around that plaquette. This Peierls idea underlies the realizations of topological lattices illustrated in Fig. 5a,c.

A network of inductors and capacitors can be assembled to realize time-reversal-invariant spin-Hall[115,116] and Weyl[117] band structures, as well as to observe higher order topological features such as corner states in quadrupole insulator lattices[118]. While this construction can be made extremely low loss and insensitive to fabrication disorder, the absence of time-reversal-symmetry breaking intrinsic to the use of just inductors and capacitors makes it less clear what many-body phases will be obtained when strong interphoton interactions are added to the system.

To overcome this limitation and break time-reversal symmetry, an ultra-low-loss topological lattice based on bulk cavities containing ferrimagnetic materials was proposed[119]. In this configuration, the modes on the individual lattice sites with opposite angular momentum are split by the magnetic element (see Fig. 5a). This technique has been employed to realize topological Chern bands in a 1/4-flux Hofstadter model[120] (Fig. 5b). The ongoing challenge is now to introduce qubits into the array and thereby realize interactions between the photons, moving from single-particle topological physics to many-body topological order.

The first experiment combining a synthetic magnetic field with strong photon–photon interactions was reported in ref. [50]. Using three superconducting qubits placed in a ring geometry (see Fig. 5c–d), the authors synthesize artificial magnetic fields by sinusoidally modulating the qubit couplings so to induce a non-trivial Peierls phase. As a signature of the time-reversal-breaking synthetic magnetic field, they observed a directional circulation of single-photon wavepackets (Fig. 5e). The interplay of the synthetic magnetic field with strong interactions was then apparent in the opposite direction of circulation of a two-photon state: when the authors inserted a pair of photons into a pair of neighbouring sites, as shown in Fig. 5f, they observed an overall rotation with opposite chirality as compared to individual photons. This striking result indicates that, as a result of strong interactions, the photons do not move freely but behave as effectively impenetrable particles.

The observed dynamics is most simply understood as the motion of a photon vacancy: akin to holes in an electronic band, the oppositely 'charged' vacancies circulate in the opposite direction compared to photons. This pioneering result paralleled the related observation of two-body quantum dynamics of ultracold bosons in a Hofstadter ladder[121]. A recent work[122] reported the observation of single-particle topological end states in a circuit magnon insulator, a configuration that automatically imposes hard-core constraint on the excitations and is thus promising in view of many-body physics.

The natural next step will be to scale up these devices to full lattice geometries, for which the possibility of realizing fractional quantum Hall states has been investigated[30,81,123] and symmetry-protected topological phases[124] are under exploration.

Flat bands and curved-space dynamics

A particular advantage of the circuit QED platform is the ability to realize complex lattice connectivities by simply changing the wiring of the circuit. Clear demonstrations of this possibility are the periodic boundary conditions that were realized for the aforementioned topological lattices in both 2D (ref. [115]) and 3D (ref. [117]) geometries. By contrast, such configurations are obviously hard if not impossible to obtain in real solid materials and require cumbersome synthetic dimensional set-ups in ultracold atomic systems[125]. Given the link between the topology of the ambient space and the degeneracy of the ground state of strongly correlated fluids displaying non-Abelian anyons, one can anticipate that experimental control on the global properties of the ambient space will play an important role in fault-tolerant quantum computing[126].

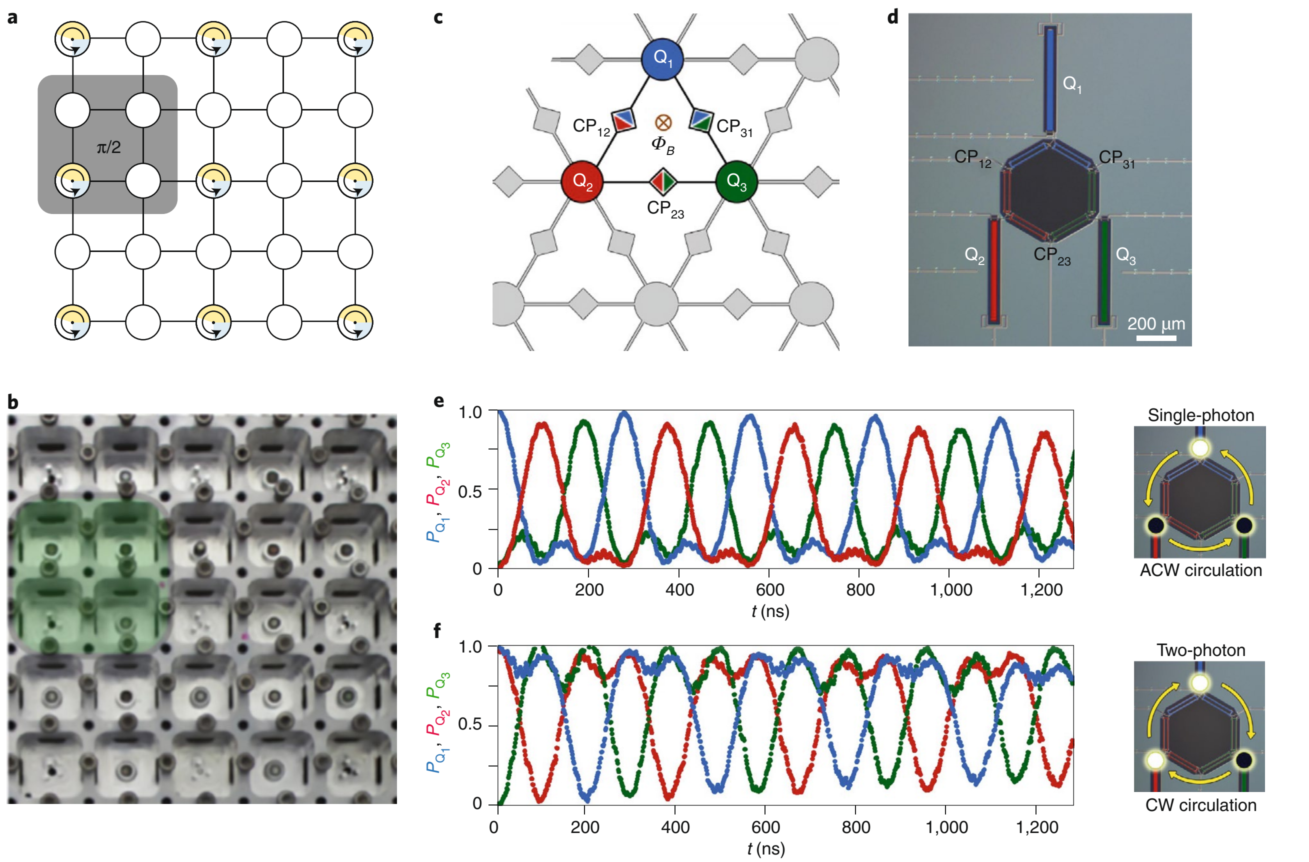

Fig. 5 | Topological lattices. Summary of two experiments demonstrating topological lattices for photons in circuit QED platforms. **a**, A schematic of a quarter-flux Hofstadter model, a model of non-interacting particles on a square lattice pierced by an orthogonal (synthetic) magnetic field. In this realization, the synthetic magnetic field is induced by enforcing that every fourth lattice site has a chiral mode function (circular arrow), which imposes the requisite tunnelling phases. All other sites (open circles) exhibit instead spatially uniform cavity mode functions. **b**, Top view of the experimental lattice, as realized by machining an array of 3D coaxial resonators. The chiral sites are implemented by introducing yttrium–iron–garnet ferrimagnets to those sites. The four-site plaquette highlighted in green is the physical realization of the four-site plaquette highlighted in gray in **a**. **c**, In the alternative approach schematically shown here, the requisite tunnelling phase is obtained by temporally modulating the coupling between adjacent lattice sites and photons are effectively impenetrable particles. A single plaquette of such a lattice is highlighted in the schematic. Φ_B, effective magnetic flux. **d**, Micrograph of the physical implementation of a three-site triangular closed loop configuration, consisting of three superconducting qubits Q_j connected via adjustable couplers CP_{jk}. **e**, Single-photon counter-clockwise circulation resulting from the time reversal breaking by the synthetic magnetic field piercing the loop. The three curves show the measured time-dependence of the probability of the photon occupying each qubit P_{Q_j} for the coherent evolution starting from a state with a single photon in Q_1. **f**. When the same experiment is performed starting with a single photon in each of Q_1 and Q_2, the strong photon–photon interactions result in circulation of an oppositely-charged vacancy in the opposite direction. Figure reproduced with permission from: **a**,**b**, ref. [120]; **c**–**f**, ref. [50], Springer Nature Ltd.

It has recently become apparent that the flexibility in the design of the lattice geometry is far broader than this. Even within the scope of planar, non-crossing designs for which fabrication is mature, it is possible to realize connectivities wherein particles explore curved spaces and non-Euclidean geometries, or experience destructive interference effects that entirely inhibit tunnelling in so-called flat bands. Such flat-band models present opportunities for new physics, as particles living in flat bands have no kinetic energy, so any interaction between the particles, in conjunction with the structure of the band, completely determines the particles' ordering[82,127,128].

While a significant suppression of the kinetic energy naturally occurs in the topological Hofstadter model discussed in the previous section, the first realization of a completely flat band in circuit QED appeared in the lattice shown in Fig. 6a. This lattice appears visually to be honeycomb, but because the resonators (depicted as light blue lines) are actually the sites and not the links, the actual lattice (shown in dark blue) that the photons inhabit (yellow dots) is kagome[11,129]. The kagome lattice is known to exhibit a flat band that is unfortunately not energetically isolated. As such any photon–photon scattering will drive photons into adjacent bands, thus it is essential to find a way to isolate the flat band.

In the quest to explore flat-band circuits, it was discovered[129] that flat bands can be isolated by exploiting a duality between sites and links: in periodic tight-binding lattices consisting of sites connected equally to three neighbours, swapping sites and links yields an identical (but shifted) band structure with a new flat band. From here, frustrating the initial lattice to minimize its bandwidth energetically isolates the flatband[129–131]. Such a flat band could be realized[48] in a lattice with heptagonal honeycomb connectivity shown in Fig. 6b, but unfortunately identical heptagons cannot tile a Euclidean plane. On the other hand, a hyperbolic plane of constant negative curvature can be tiled with heptagons. The arbitrary connectivity and the decoupling of geometrical arrangement and hopping rate possible in circuit QED systems enable such a lattice to be implemented in a planar circuit as shown experimentally in Fig. 6c. Recent theoretical breakthroughs provide a general path to planar lattices with isolated flatbands[128].

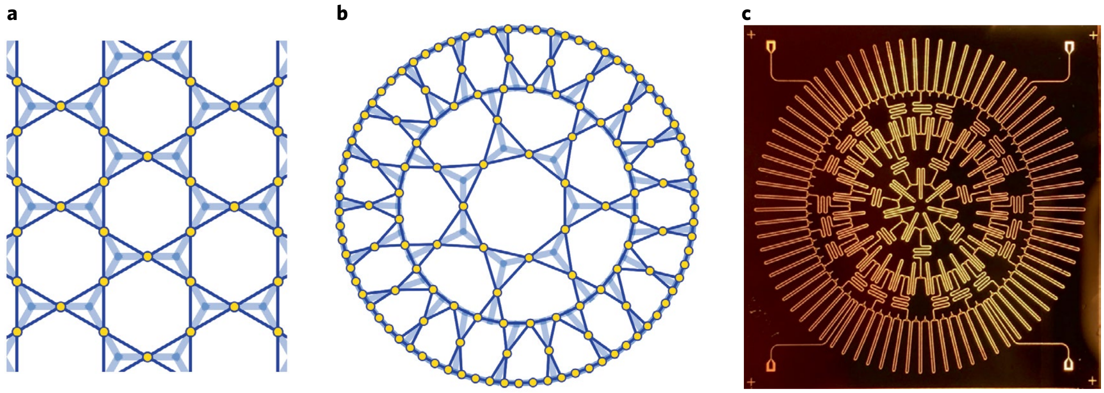

Fig. 6 | Curved-space and flat-band lattices. a, If a honeycomb lattice is realized by using resonators as the links (light blue), the photons inhabiting those resonators tunnel around in a tight-binding lattice which is effectively kagome (dark blue) because the 'sites' of this lattice are the resonators themselves. **b**, Equivalent overlay for a heptagonal-honeycomb lattice. If distances are to be preserved, this lattice can only be embedded in a non-Euclidean hyperbolic plane. **c**, Photograph of a resonator lattice which realizes the heptagonal-honeycomb lattice of **b**: different resonators have different physical shapes to compensate for the variable physical lengths of the bonds. Figure reproduced with permission from: **c**, ref. [48], Springer Nature Ltd.

Once again, the natural next step is to add strong interactions to these curved-space models. It is natural to expect that particles inhabiting curved surfaces have their ordering perturbed by the Gaussian curvature, which introduces pleats[132] and other topological defects much as a full head of hair must have a cowlick[133]. Furthermore, it was recently discovered that the celebrated Laughlin wavefunction has a curved-space generalization[134], whose response to spatial curvature reveals new topological quantum numbers[135].

In general, the inherently higher connectivity of hyperbolic spaces compared to their Euclidean counterparts, as well as realization of higher-dimensional lattices via the synthetic dimension concept[136], is anticipated to have a profound effect on many-body physics, enhancing the effects of interactions and frustration, speeding up the propagation of entanglement, and substantially modifying surface-tension-like effects governing phase boundaries and domain growth[137]. These features are especially promising in view of topological quantum computing applications, where they are expected to improve performance of surface codes[138].

Looking ahead

The photonic quantum matter effort has taught us how to create customized 'particles' from the ground up, to cool them, and to characterize the microscopic structure of the resulting ordering. In the process, we have gained a unique perspective on what makes a material a material. In the quest to study new phases of matter, the work of realizing the ingredients often reveals exciting physics of its own: we have seen that exploring curved-space physics yields new insights into flat bands; stabilizing a Mott insulator poses new questions about dissipatively driven steady states; spectroscopic probes of many-body localization provide new insights into level spacings in the localized phase. The quantum-matter frontier holds surprises around every corner, each awaiting discovery with the unique probes and understanding afforded by the various platforms.

Despite this explosion of progress and innovation in the circuit materials community, it is clear that we are closer to the beginning of this journey than to its end. As we have seen, the first realizations of strongly correlated photonic matter have only just come online, and have yet to be integrated at any scale with the zoology of topological and exotic lattice models outlined in the latter parts of this work. In a similar vein, the toolset to dissipatively stabilize strongly correlated matter has now been realized in its minimal form, but has yet to be applied to any but the simplest Mott models, or formally connected to thermodynamics. Given the flexibility of the circuit QED platform in creating new lattices and new interactions, we can reasonably expect that new quantum phases of matter are waiting to be discovered, with a number of potential applications to quantum technologies.

As in the last decade, technical innovation will remain a driver of the field, and we expect to benefit richly from ongoing circuit quantum computing work. Advances are essential on two fronts: (1) reduction of the fabrication disorder of superconducting qubits: current fabrication techniques produce large (5–10%) variation in qubit frequencies, and while site-by-site flux tuning is possible[42], suppressing qubit disorder to the $\sim 2 \times 10^{-4}$ level achieved for resonators[47] would be transformative for scaling; and (2) overcoming the tyranny of planar connectivity: at present, most research groups implement circuits on single-sided sapphire wafers, limiting the achievable graph connectivity. Large-scale incorporation of air bridges[139], bump bonds[140], or superconducting vias[141] will enable fully arbitrary connectivity between lattice sites.

Opportunities loom to explore yet-more-exotic models, where photons interact with one another at range[142,143] or via more complex potentials, which are anticipated to lead to fractal band structures[144], intriguing topological orders[145], or even models of bosons coupled to dynamical gauge fields[146]. While this physics is in principle accessible given existing qubit topologies with appropriately engineered drives, one of the unique strengths of the circuit QED platform is the ability to engineer and fabricate new fundamental building blocks with exotic properties. It seems clear that such engineering, while intellectually expensive on the front-end, will lead to simpler and more robust tools that scale more manageably.

In terms of experimental opportunities, a challenge immediately facing the community is to realize bosonic analogues of the fractional quantum Hall states[30,123], which extend the recent pioneering demonstration in spatially continuous geometries[60] to discrete lattice configurations. While photonic topological materials will likely never exist with particle number comparable to their electronic counterparts, numerics suggest[18,147–150] that much of the phenomenology that a material exhibits emerges from the microscopic quantum dynamics of a few interacting particles. As such, there are fascinating opportunities to stabilize small Laughlin puddles of light in topological lattices that scale up to a full-fledged 2D lattice by building upon the trimer demonstrated in ref. [50] or by combining the topological lattice described in ref. [120] with the interactions demonstrated in ref. [42].

Dissipative stabilization will be a crucial ingredient of any such topological material, though the slow motion of quasi-holes in a Laughlin fluid will likely demand distributed stabilizers, and their fractional charge will likely affect the refilling efficiency[36,81]. From here, the reward to be reaped in the mid-term is the long-awaited experimental verification of the anyonic statistics of the quasi-particle excitations of quantum Hall fluids[145,151–154], an exciting concept that has so far eluded conclusive observation in traditional electronic systems[126,155] and is presently under active study in circuit QED platforms from the alternate point of view of toric codes[156]. To this and related purposes, the artillery of manipulation and detection techniques as well as the high repetition rate of circuit QED systems will allow for measurements of fundamental many-body quantities such as correlation functions and Green functions with unprecedented statistics, giving game-changing information on the microscopic properties of topological matter.

A long-run objective with potentially revolutionary technological applications will then be to generate more complex many-body states that support non-Abelian anyons and to demonstrate their exotic statistics under exchange. Here, the great challenge is to take advantage of the local nature of loss and pumping processes to extend the concept of topological protection of the state to the driven-dissipative context. In this way, one could encode quantum information in a topologically degenerate many-body state of light in a fault-tolerant way and exploit anyon exchange to perform quantum computations[126].

From a fundamental perspective, the dynamics of strongly interacting photons as they thermalize in any relatively large lattice system, topological, exotic or otherwise, proffers fascinating questions[157]: To what extent do the dynamical steady-states resulting from dissipative stabilization mirror their equilibrium counterparts[94] and how does the non-equilibrium nature affect phase transitions and critical behaviour[24,90,96–99]? Does this depend crucially on the precise nature of the drive or are there universal features? Is it possible to develop coarse-grained hydrodynamic models that capture the essential behaviour of these systems[158]? How would such models depend upon the microscopic parameters of the underlying physical platform? Although these questions may appear specific to the circuit QED platform, they strike at the very heart of quantum many-body physics: how do the laws of the macroscopic classical world emerge from the underlying quantum froth[159]?

It is interesting to ask how digital quantum computers might supplement custom-built synthetic quantum materials for basic condensed-matter research. Through Trotterization (decomposing an arbitrary operator into easily implemented operators through the Trotter–Suzuki formula), a digital quantum computer can simulate nearly any Hamiltonian system. Without error correction, however, Trotterization is quite inefficient in its use of the accessible quantum coherence. The extraordinary flexibility of circuit-based quantum materials to directly engineer the physics of interest allows them to make more efficient use of coherence and, in many cases, exhibit higher tolerance to errors. This makes them an attractive way to explore high entanglement-depth quantum dynamics of exotic many-body systems.

It is thus apparent that circuit QED provides a wholly new way to look at matter. This novel perspective has already provided new understandings of problems from dynamics in curved space and topological physics to quantum thermalization and many-body localization. The array of rich science emerging from recent endeavors notwithstanding, this lively field has posed many more questions than it has answered: we anticipate that the coming years will furnish answers to these questions, and allow the community to investigate yet-more-fundamental questions at the interfaces of quantum many-body physics, the emergence of the classical properties from entanglement in driven-dissipative quantum systems, and quantum information science.

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Competing interests
The authors declare no competing interests.

Additional information
Correspondence should be addressed to J.S.